solved examples of fourier series

solved examples of fourier series provide an essential understanding of how periodic functions can be expressed as an infinite sum of sine and cosine terms. This mathematical tool is widely used in engineering, physics, and applied mathematics to analyze signals, solve differential equations, and model periodic phenomena. Through various solved examples of Fourier series, one can grasp the process of determining Fourier coefficients, the convergence of series, and the practical applications of these expansions. This article explores several illustrative examples, starting from simple piecewise functions to more complex periodic waveforms. Additionally, the step-by-step solutions will enhance comprehension of key concepts such as even and odd functions, half-range expansions, and the use of Euler's formula. The following sections will guide readers through fundamental to intermediate examples, reinforcing theoretical knowledge with practical computations.

Basic Solved Example of Fourier Series

Understanding the Fourier series begins with analyzing simple periodic functions, such as the function f(x) = x defined over the interval [-π, π]. This example illustrates the computation of the Fourier coefficients and the construction of the series representation. The function is odd, which simplifies the Fourier series to sine terms only, highlighting the connection between function symmetry and the Fourier expansion.

Finding Fourier Coefficients for f(x) = x

Given the function f(x) = x on the interval [-π, π], which is periodic with period 2π, the goal is to express f(x) as a Fourier series:

$f(x) = a_0/2 + \Sigma (a_n \cos nx + b_n \sin nx)$, n=1 to ∞.

Since f(x) is odd, all cosine coefficients a_n and the constant term a_0 are zero. The sine coefficients b_n are calculated as:

$b_n = (1/\pi)$ ∫ from -π to π f(x) sin(nx) dx

Using the odd function property, the integral simplifies to:

$b_n = (2/\pi) \int$ from 0 to π $x \sin(nx)\, dx$

Evaluating this integral by parts yields:

$b_n = (2/\pi) [(-x \cos(nx))/n + (\sin(nx))/n^2]$ from 0 to π = $(2/\pi) ((-\pi \cos(n\pi))/n + (\sin(n\pi))/n^2 - 0) = (2/\pi)(-\pi (-1)^n / n) = 2 (-1)^{n+1} / n$.

Thus, the Fourier series for $f(x) = x$ is:

$f(x) = 2 \Sigma (-1)^{n+1} (\sin nx) / n, n=1$ to ∞.

Fourier Series of a Square Wave

The square wave is a classic example in Fourier analysis, demonstrating how a discontinuous periodic function can be represented by a convergent trigonometric series. Its Fourier series captures the waveform’s shape through an infinite sum of sine functions with odd harmonics.

Definition and Setup of the Square Wave

Consider the square wave function defined on $[-\pi, \pi]$ as:

- $f(x) = 1$ for $0 < x < \pi$
- $f(x) = -1$ for $-\pi < x < 0$

This function is odd and periodic with period 2π. The Fourier series will only contain sine terms due to the odd symmetry.

Calculation of Fourier Coefficients

The sine coefficients b_n are given by:

$b_n = (1/\pi) \int$ from $-\pi$ to π $f(x) \sin(nx)\, dx$

Since $f(x)$ is odd, simplify the integral over $[0, \pi]$:

$b_n = (2/\pi) \int$ from 0 to π $\sin(nx)\, dx = (2/\pi) [-\cos(nx)/n]$ from 0 to π = $(2/\pi) (1 - \cos(n\pi))/n$.

Because $\cos(n\pi) = (-1)^n$, b_n is zero for even n and nonzero for odd n:

$b_n = (4)/(n\pi)$ for n odd, and $b_n = 0$ for n even.

Therefore, the Fourier series is:

$f(x) = (4/\pi) \Sigma (\sin((2k - 1)x) / (2k - 1)), k=1$ to ∞.

Half-Range Fourier Series Expansions

Half-range expansions are used when a function is defined only on a half-interval, such as $[0, L]$. These expansions extend the function to $[-L, L]$ either as an even or odd function, resulting in cosine or sine Fourier series respectively. This technique is useful for solving boundary value problems with specific conditions.

Half-Range Cosine Series Example

Consider f(x) = x defined on [0, π]. Extending f(x) as an even function over [-π, π] allows representation using only cosine terms:

The cosine coefficients are:

$a_0 = (2/\pi) \int$ from 0 to π f(x) dx,

$a_n = (2/\pi) \int$ from 0 to π f(x) cos(nx) dx.

Calculations yield:

- $a_0 = \pi$
- $a_n = (2/\pi) [(\pi (-1)^n)/n^2]$

The half-range cosine series becomes:

$f(x) = \pi/2 + \Sigma (2 (-1)^n / n^2) \cos(nx)$, n=1 to ∞.

Half-Range Sine Series Example

Alternatively, extending f(x) = x as an odd function over [-π, π] results in a sine-only Fourier series identical to the full-range odd extension. This approach simplifies solving problems with zero boundary conditions at x=0.

Fourier Series for Piecewise Functions

Piecewise functions, which have different definitions over subintervals, are common in practical applications. Deriving their Fourier series involves integrating over each piece separately and combining the results carefully to compute coefficients.

Example: Piecewise Linear Function

Define f(x) on [-π, π] as:

- f(x) = 0 for $-\pi \le x < 0$
- f(x) = x for $0 \le x \le \pi$

This function is neither even nor odd, so both sine and cosine terms appear in its Fourier series.

Computing Fourier Coefficients

The coefficients are computed by splitting the integrals:

- $a_0 = (1/\pi)$ [∫ from $-\pi$ to 0 0 dx + ∫ from 0 to π x dx] = $(1/\pi)(\pi^2 / 2) = \pi/2$
- $a_n = (1/\pi)$ [∫ from $-\pi$ to 0 0 cos(nx) dx + ∫ from 0 to π x cos(nx) dx]
- $b_n = (1/\pi)$ [∫ from $-\pi$ to 0 0 sin(nx) dx + ∫ from 0 to π x sin(nx) dx]

Evaluating these integrals by parts produces explicit expressions for a_n and b_n, allowing reconstruction of the Fourier series for f(x).

Applications and Interpretations of Fourier Series

Fourier series, demonstrated through solved examples, play a vital role in signal processing, heat conduction, vibration analysis, and more. Understanding the convergence properties and the physical meaning of coefficients aids in interpreting complex periodic phenomena.

Key Applications Illustrated by Examples

1. Signal Decomposition: Breaking down complex signals into fundamental frequencies.
2. Heat Equation Solutions: Using Fourier series to solve PDEs with boundary conditions.
3. Electrical Engineering: Analyzing AC circuits and waveform distortions.
4. Mechanical Vibrations: Modeling oscillations in structures and machinery.

Interpretation of Fourier Coefficients

The magnitude of Fourier coefficients indicates the contribution of each harmonic frequency to the original function. Rapidly decreasing coefficients correspond to smoother functions, while slower decay signifies discontinuities or sharp transitions, as shown in the square wave example.

Frequently Asked Questions

What is a Fourier series and how is it used in solving problems?

A Fourier series is a way to represent a periodic function as a sum of sine and cosine terms. It is used to analyze and solve problems involving periodic functions by breaking them down into simpler trigonometric components.

Can you provide a simple solved example of a Fourier series for a basic function?

Yes. For example, the Fourier series of the function $f(x) = x$ on the interval $(-\pi, \pi)$ is given by $f(x) = 2(\sum_{n=1}^{\infty} (-1)^{n+1} / n * \sin(nx))$. This series converges to the function for all x in $(-\pi, \pi)$.

How do you find the Fourier coefficients in a solved example?

To find Fourier coefficients, you use the formulas: $a_0 = (1/2\pi)\int_{-\pi}^{\pi} f(x)\, dx$, $a_n = (1/\pi)\int_{-\pi}^{\pi} f(x) \cos(nx)\, dx$, and $b_n = (1/\pi)\int_{-\pi}^{\pi} f(x) \sin(nx)\, dx$. Integrating the given function multiplied by sine or cosine over one period yields the coefficients.

What is a solved example of Fourier series for a piecewise function?

Consider the function $f(x) = \{0 \text{ for } -\pi < x < 0, 1 \text{ for } 0 \leq x < \pi\}$. Its Fourier series solution involves calculating coefficients a_n and b_n using integrals over each piece, resulting in a series that converges to the function almost everywhere.

How do solved examples of Fourier series help in engineering applications?

Solved examples of Fourier series illustrate how complex periodic signals can be decomposed into simpler sinusoidal components. This helps engineers in signal processing, heat transfer, vibrations, and electrical circuits by providing analytical solutions and insights into system behavior.

Additional Resources

1. *Fourier Series and Boundary Value Problems*
This book provides a thorough introduction to Fourier series with numerous solved examples that illustrate the application of these series to boundary value problems. It covers fundamental concepts, orthogonality of functions, and convergence issues, making it ideal for students and engineers. The solved problems demonstrate practical applications in physics and engineering, enhancing conceptual understanding.

2. *Fourier Series: A Modern Introduction*

Designed for beginners and intermediate learners, this title offers a comprehensive exploration of Fourier series with step-by-step solutions to classical problems. The book emphasizes problem-solving strategies and includes detailed examples that clarify the computation of coefficients and convergence criteria. It also introduces applications in heat transfer and signal processing.

3. *Applied Fourier Analysis with Solved Examples*
Focusing on the practical application of Fourier analysis, this book contains a rich collection of solved problems on Fourier series. It bridges theory and practice by demonstrating how Fourier series can be used to solve differential equations and analyze periodic functions. The clear explanations and worked-out examples support self-study and reference.

4. *Fourier Series and Transforms: A Problem-Solving Approach*
This text integrates theory with a problem-solving methodology, providing a large number of solved examples that help readers master Fourier series and transforms. It thoroughly explains convergence, Parseval’s theorem, and Gibbs phenomenon through practical problems. This book is suitable for advanced undergraduates and graduate students in applied mathematics and engineering.

5. *Introduction to Fourier Series with Worked Examples*
A user-friendly introduction that focuses on building intuition for Fourier series through numerous worked examples. The book clearly explains the derivation of Fourier coefficients and the application of series to periodic functions. Its practical approach makes it an excellent resource for students encountering Fourier series for the first time.

6. *Fourier Series and Orthogonal Functions: Examples and Applications*
This book emphasizes the role of orthogonal functions in Fourier series expansions, offering a variety of solved problems that illustrate these concepts. It covers classical Fourier series, generalized Fourier series, and their applications in engineering and physics. The examples help readers understand how orthogonality simplifies the representation of functions.

7. *Problems and Solutions in Fourier Series and Partial Differential Equations*
A problem-centric book that presents a wide range of solved examples combining Fourier series with partial differential equations. It includes classical problems in heat conduction, wave equations, and Laplace’s equation, with detailed solution steps. The book is designed to enhance problem-solving skills and reinforce theoretical knowledge.

8. *Fourier Series: Theory and Solved Problems*
This title provides a balanced treatment of theoretical foundations and practical problem-solving in Fourier series. It contains numerous solved examples that cover convergence tests, coefficient calculations, and applications to signal analysis. The clear and concise explanations make it a valuable reference for both students and professionals.

9. *Fourier Series Made Easy: Step-by-Step Examples*
A concise guide aimed at simplifying the learning process of Fourier series through step-by-step solved problems. The book breaks down complex topics into manageable parts and includes examples related to engineering and physics applications. It is particularly useful for self-learners seeking clarity and practical understanding.

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