

trigonometric problems with solutions and answers

Trigonometric problems with solutions and answers are an essential part of mathematics, especially in fields like physics, engineering, and computer science. Trigonometry, derived from the Greek words for "triangle" and "measurement," focuses on the relationships between the angles and sides of triangles. This article will delve into various types of trigonometric problems, providing step-by-step solutions and answers to help you understand these concepts better.

Understanding Trigonometric Functions

Before we dive into specific problems, it's crucial to understand the basic trigonometric functions:

- Sine (sin): The ratio of the length of the opposite side to the hypotenuse in a right triangle.
- Cosine (cos): The ratio of the length of the adjacent side to the hypotenuse.
- Tangent (tan): The ratio of the length of the opposite side to the adjacent side, or $\tan(\theta) = \sin(\theta)/\cos(\theta)$.

These functions can be represented in terms of a right triangle or on the unit circle, where the radius is 1.

Basic Trigonometric Identities

Before solving problems, familiarize yourself with these fundamental identities:

1. Pythagorean Identity:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

2. Reciprocal Identities:

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)}$$

3. Quotient Identities:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

4. Co-Function Identities:

$$\sin(90^\circ - \theta) = \cos(\theta)$$

$$\tan(90^\circ - \theta) = \cot(\theta)$$

5. Even-Odd Identities:

$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

Trigonometric Problems with Solutions

Now, let's look at some trigonometric problems and their solutions.

Problem 1: Find the Value of $\sin(30^\circ)$

Solution:

To find $\sin(30^\circ)$, recall that on the unit circle, the sine of an angle is the y-coordinate of the corresponding point.

$$\sin(30^\circ) = \frac{1}{2}$$

$$\text{Answer: } \sin(30^\circ) = \frac{1}{2}$$

Problem 2: Solve for x in the equation $2\sin(x) = \sqrt{3}$ for $0^\circ \leq x < 360^\circ$

Solution:

1. Divide both sides by 2:

$$\sin(x) = \frac{\sqrt{3}}{2}$$

2. Find the angles where sine equals $\frac{\sqrt{3}}{2}$:

$$x = 60^\circ$$

$x = 120^\circ$ (since sine is positive in the first and second quadrants)

$$\text{Answer: } x = 60^\circ, 120^\circ$$

Problem 3: Prove the Identity $1 + \tan^2(\theta) = \sec^2(\theta)$

Solution:

Using the Pythagorean identity:

1. Start with the identity:

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

2. Square both sides:

$$\tan^2(\theta) = \frac{\sin^2(\theta)}{\cos^2(\theta)}$$

3. Add 1:

$$1 + \tan^2(\theta) = 1 + \frac{\sin^2(\theta)}{\cos^2(\theta)}$$

4. Finding a common denominator:

$$= \frac{\cos^2(\theta) + \sin^2(\theta)}{\cos^2(\theta)}$$

5. Using the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$:

$$= \frac{1}{\cos^2(\theta)} = \sec^2(\theta)$$

Answer: The identity is proven.

Problem 4: Find all solutions to the equation $\cos(2x) = 0.5$ for $0^\circ \leq x < 360^\circ$

Solution:

1. First, we recognize that $\cos(2x) = 0.5$ implies $2x = 60^\circ$ or $2x = 300^\circ$ (since cosine is positive in the first and fourth quadrants).

2. Solve for x :

- From $2x = 60^\circ$:

- $x = 30^\circ$

- From $2x = 300^\circ$:

- $x = 150^\circ$

- We also need to consider the periodicity of cosine:

- $2x = 60^\circ + 360^\circ k$ and $2x = 300^\circ + 360^\circ k$, where k is an integer.

3. Dividing by 2 gives:

- $x = 30^\circ + 180^\circ k$ and $x = 150^\circ + 180^\circ k$

4. For $k = 0$:

- $x = 30^\circ, 150^\circ$

5. For $k = 1$:

- $x = 210^\circ, 330^\circ$

Answer: The solutions are $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$.

Problem 5: Calculate the height of a tree if the angle of elevation from a point 50 meters away from the base of the tree is 30° .

Solution:

1. We can use the tangent function, which relates the angle to the opposite side (height of the tree) and the adjacent side (distance from the tree):

- $\tan(30^\circ) = \frac{\text{height}}{50}$

2. From trigonometric tables, we know:

- $\tan(30^\circ) = \frac{1}{\sqrt{3}}$

3. Set up the equation:

- $\frac{1}{\sqrt{3}} = \frac{\text{height}}{50}$

4. Solve for height:

- $\text{height} = 50 \cdot \frac{1}{\sqrt{3}}$

- $\text{height} = \frac{50}{\sqrt{3}} \approx 28.87$ meters.

Answer: The height of the tree is approximately 28.87 meters.

Conclusion

In this article, we explored various trigonometric problems with solutions and answers. Understanding these concepts is vital for students and professionals alike, as trigonometry plays a crucial role in various

applications such as physics, engineering, and architecture. Whether you're solving basic trigonometric equations or proving identities, practice and familiarity with the fundamental identities and functions will enhance your problem-solving skills. Continue to practice and apply these concepts, and you will find success in mastering trigonometry!

Frequently Asked Questions

What is the value of $\sin(30^\circ)$ and how can it be derived?

The value of $\sin(30^\circ)$ is $1/2$. It can be derived using the unit circle or right triangle properties, where in a 30-60-90 triangle, the opposite side to the 30° angle is half the hypotenuse.

How do you solve for x in the equation $\sin(x) = 0.5$?

To solve $\sin(x) = 0.5$, we find the angles for which this is true. The solutions in the interval $[0, 360^\circ]$ are $x = 30^\circ$ and $x = 150^\circ$.

What is the cosine of 45° and how is it calculated?

The cosine of 45° is $\sqrt{2}/2$. This can be calculated using the properties of a 45-45-90 triangle, where both legs are equal.

How do you convert radians to degrees for the angle $\pi/4$?

To convert radians to degrees, multiply by $180/\pi$. For $\pi/4$, $(\pi/4) (180/\pi) = 45^\circ$.

What is the solution to the equation $\tan(x) = \sqrt{3}$?

The solutions for $\tan(x) = \sqrt{3}$ in the interval $[0, 360^\circ]$ are $x = 60^\circ$ and $x = 240^\circ$.

How do you find the exact value of $\cos(120^\circ)$?

The exact value of $\cos(120^\circ)$ is $-1/2$. This can be found by recognizing that 120° is in the second quadrant where cosine values are negative.

What is the Pythagorean identity involving $\sin$ and $\cos$?

The Pythagorean identity is $\sin^2(x) + \cos^2(x) = 1$. This identity holds true for all values of x .

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